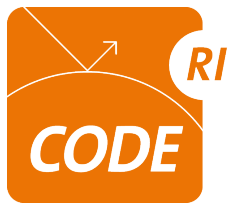
A Generic Framework for Verified Compilers Using Isabelle/HOL's Locales

Martin Desharnais Stefan Brunthaler



Research Institute
Cyber Defence
Universität der Bundeswehr München

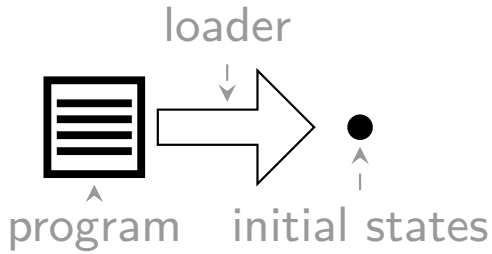
Isabelle Workshop 2020

30th June 2020

Language semantics

program

Language semantics



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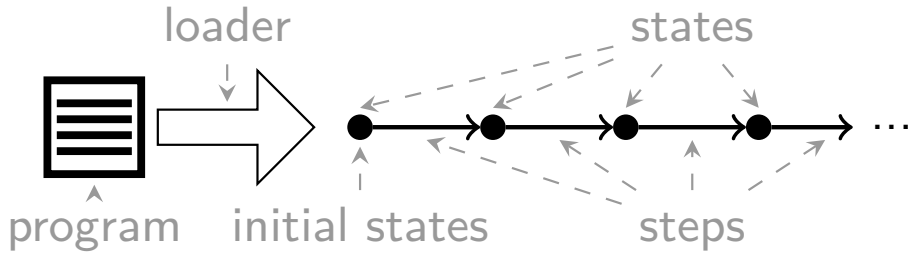

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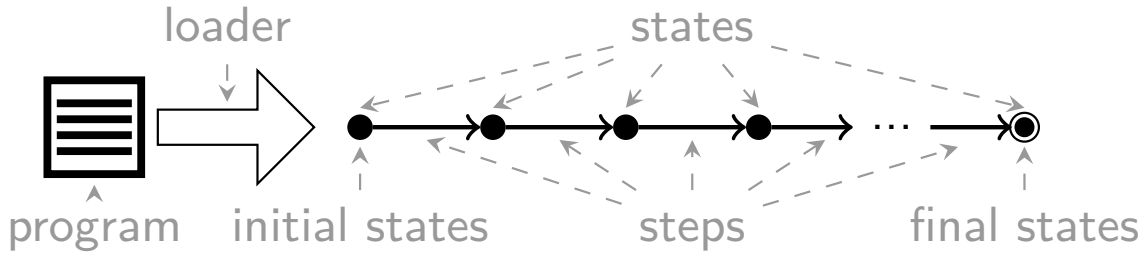

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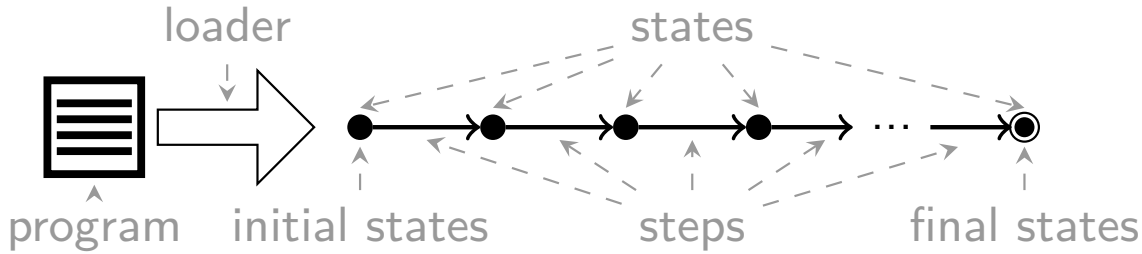

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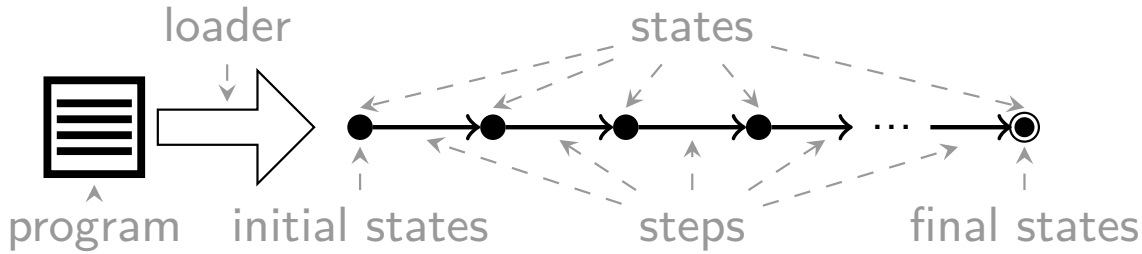

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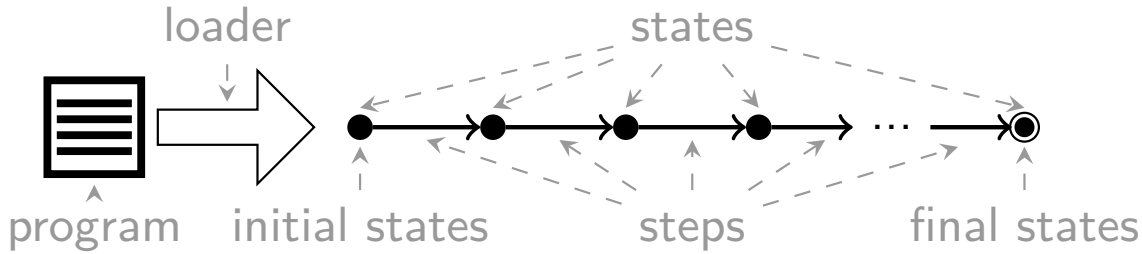

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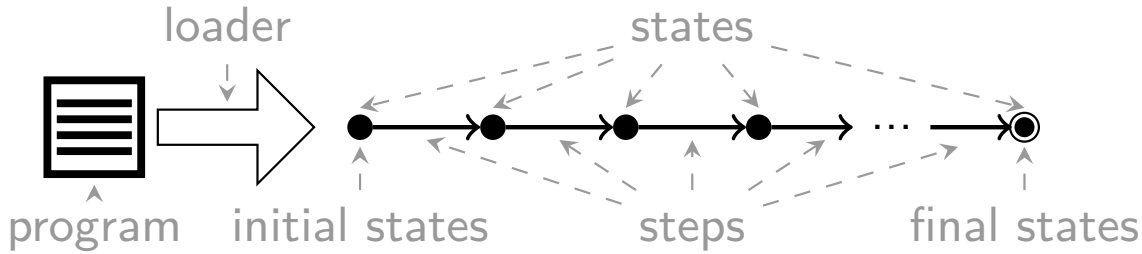

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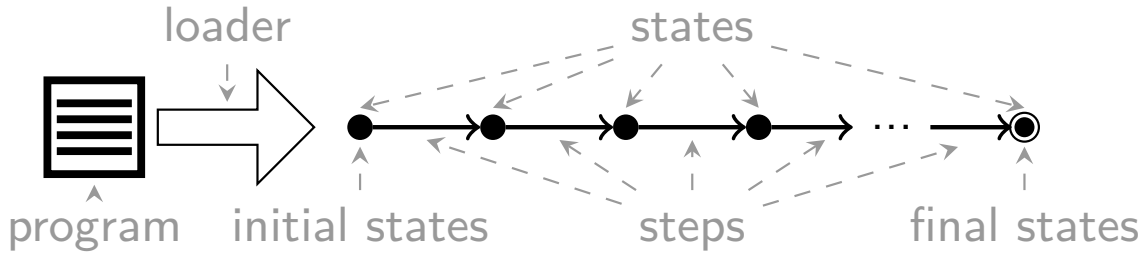

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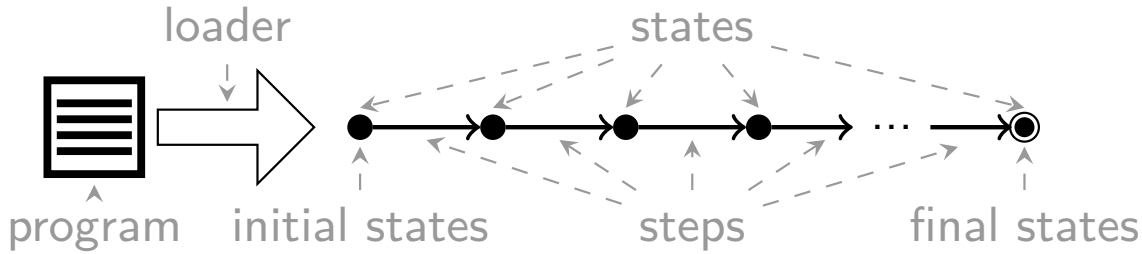
Language semantics



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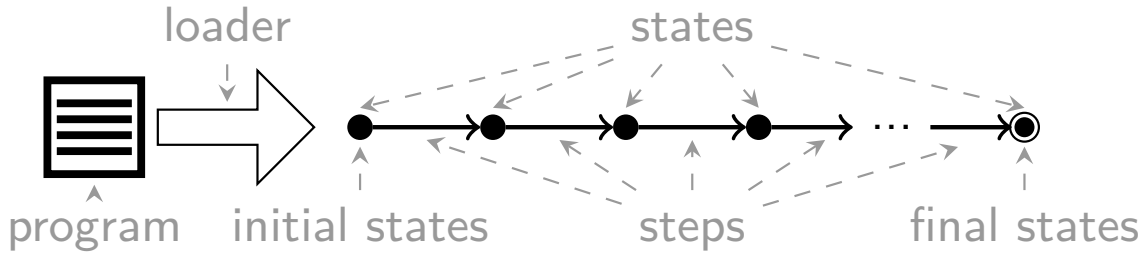
Language semantics



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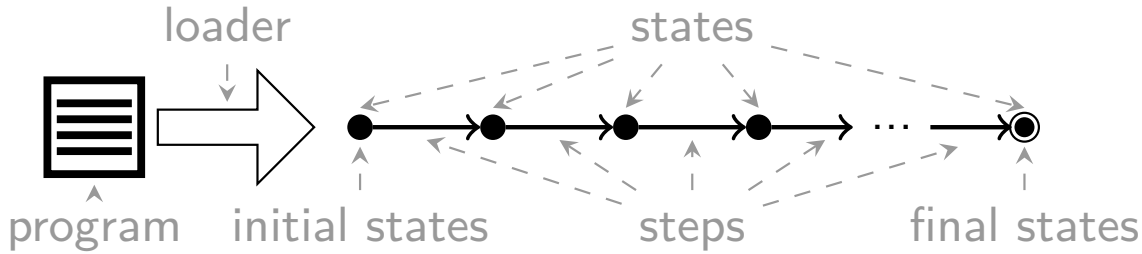

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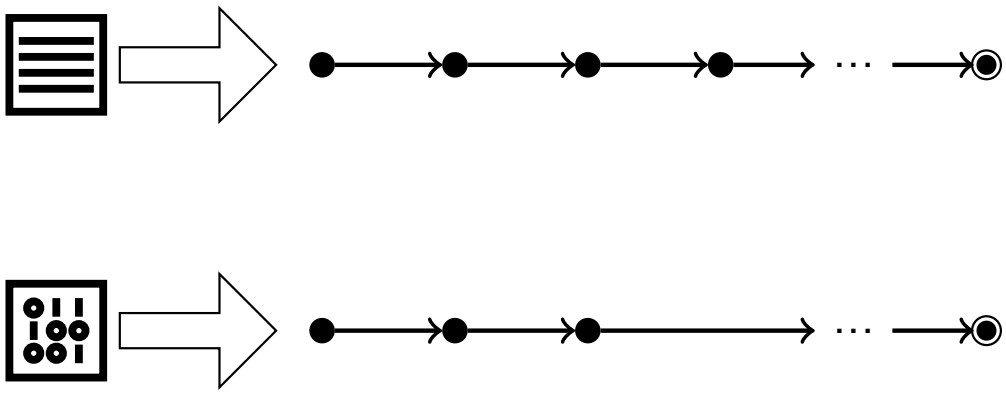
new parameters

Simulation



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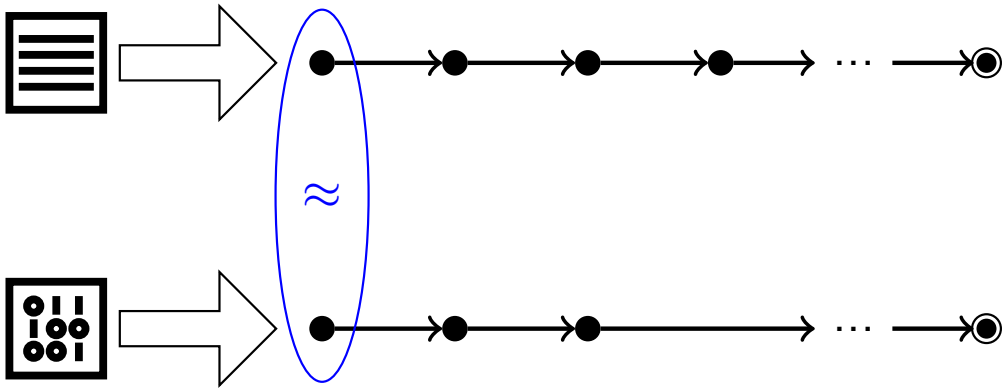

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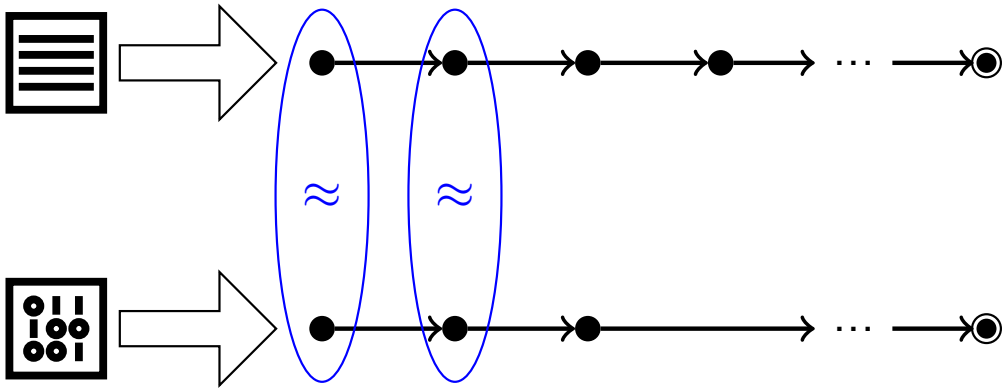

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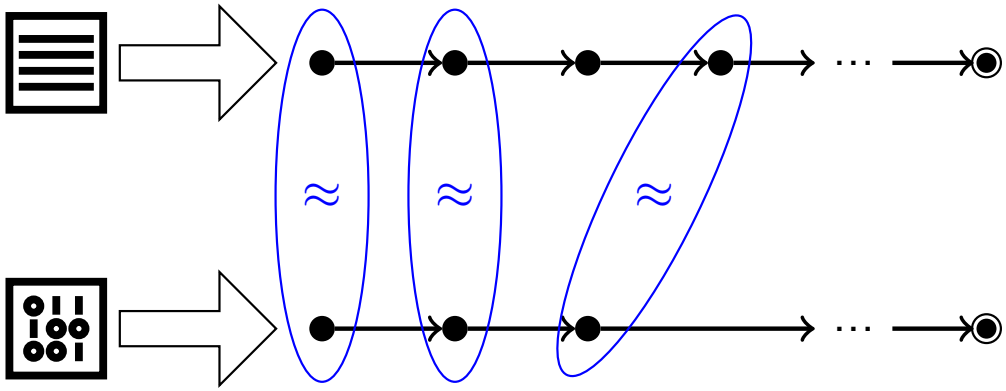

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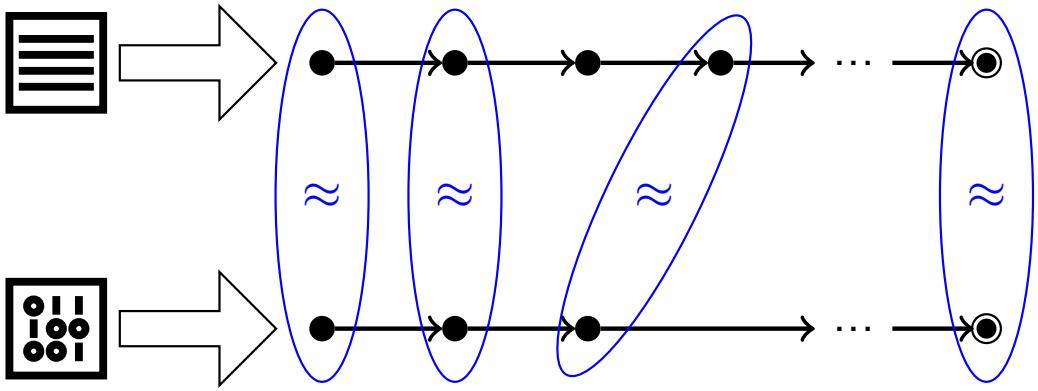

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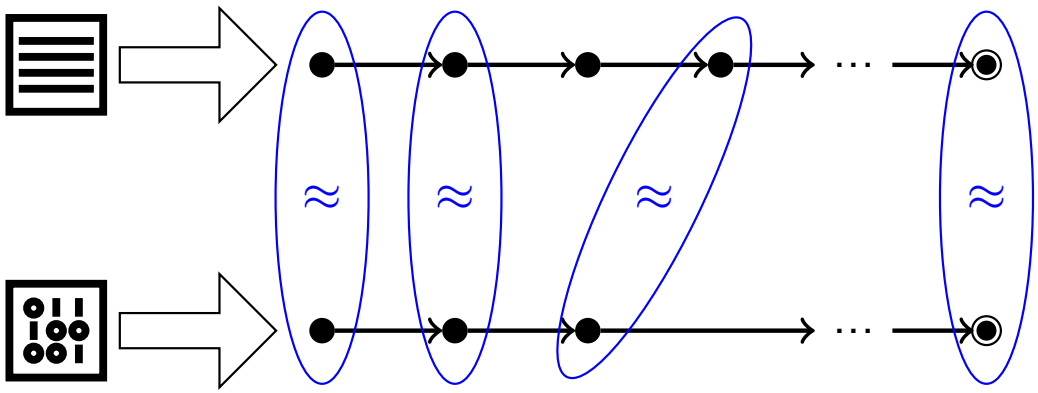

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  well-founded $\sqsubset$ **for**
    $\rightarrow_1$ **and** $\circledcirc_1$ :: *'state*$_1$ $\Rightarrow$ bool **and**
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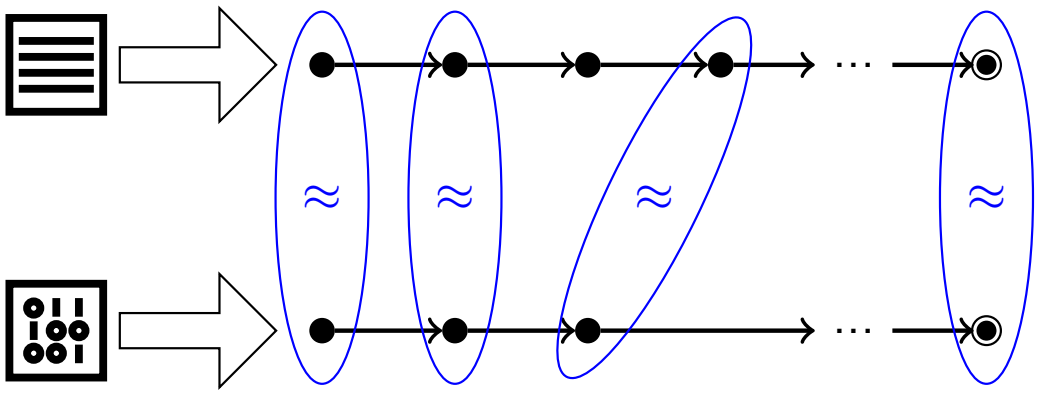

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multiple instances

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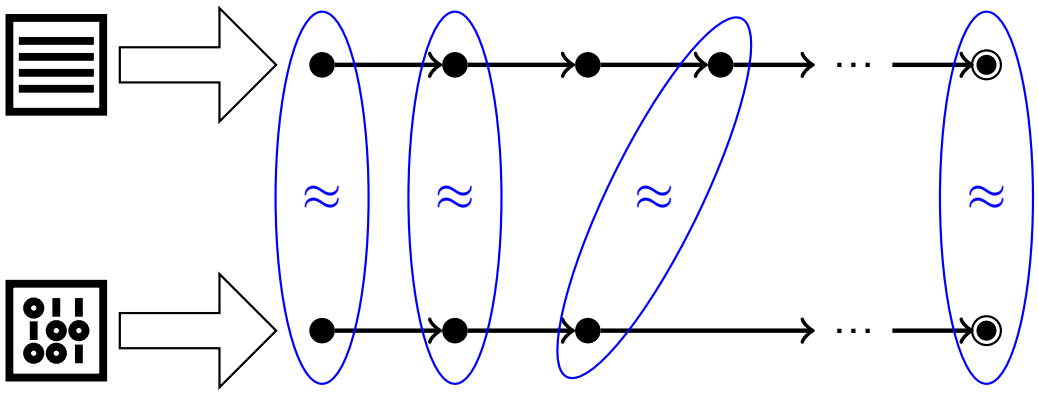

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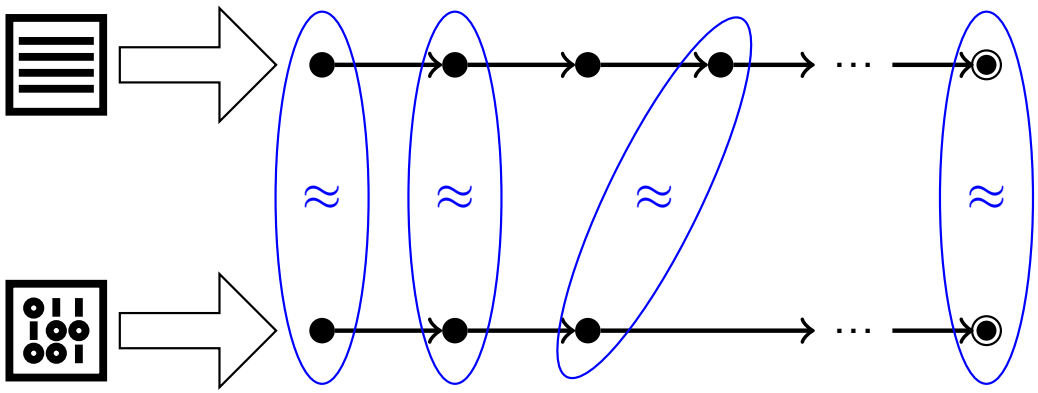

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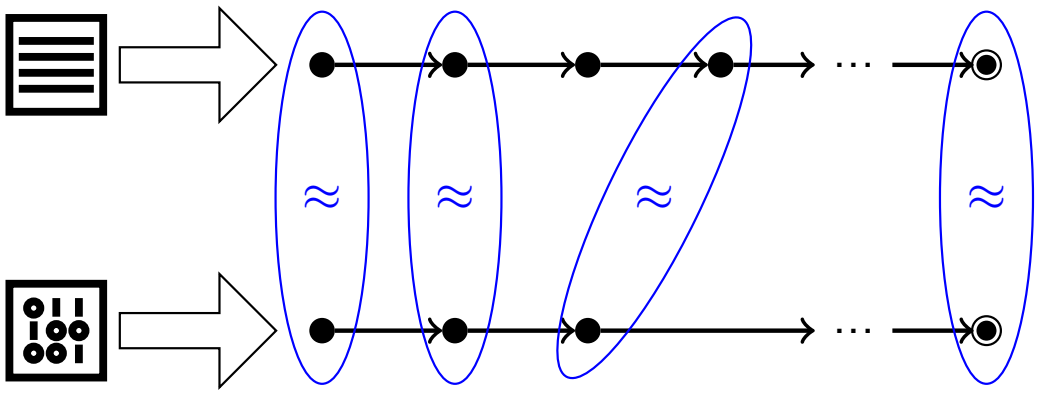

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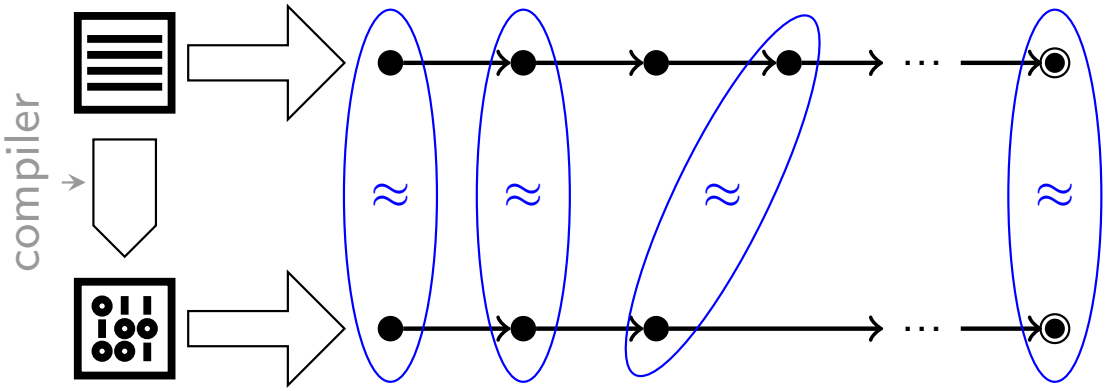
transitive closure

Compiler



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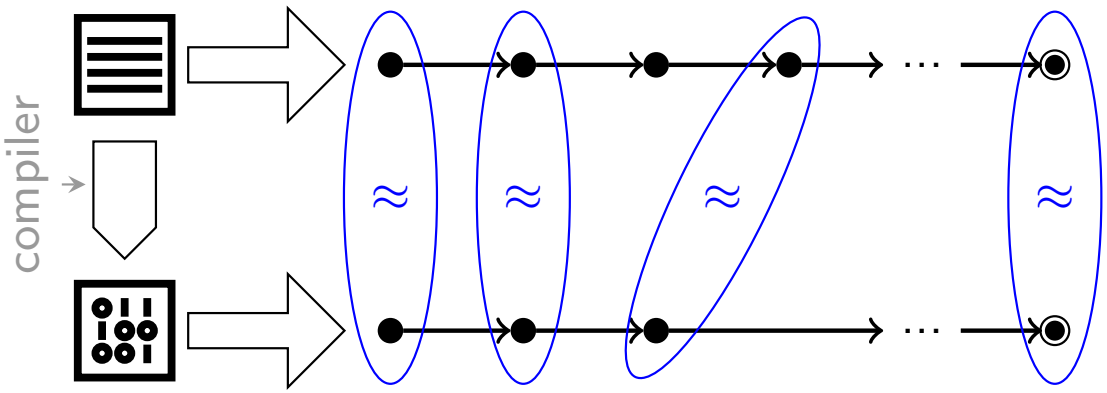

locale compiler =
  L1: language $\rightarrow_1$ $\bullet_1$ load$_1$ +
  L2: language $\rightarrow_2$ $\bullet_2$ load$_2$ +
  backward-simulation $\rightarrow_1$ $\bullet_1$ $\rightarrow_2$ $\bullet_2$ $\sqsubset$ match **for**
    $\rightarrow_1$ **and** $\bullet_1$ **and** load$_1$ :: *'prog*$_1$ $\Rightarrow$ *'state*$_1$ $\Rightarrow$ bool **and**
    $\rightarrow_2$ **and** $\bullet_2$ **and** load$_2$ :: *'prog*$_2$ $\Rightarrow$ *'state*$_2$ $\Rightarrow$ bool **and**
    $\sqsubset$ **and** match :: *'index* $\Rightarrow$ *'state*$_1$ $\Rightarrow$ *'state*$_2$ $\Rightarrow$ bool
  fixes compile :: *'prog*$_1$ $\Rightarrow$ *'prog*$_2$ option
  assumes
    compile-load: compile p_1 = Some p_2 $\Longrightarrow$ load$_1$ p_1 = Some s_1 $\Longrightarrow$
      $\exists s_2\ i.$ load$_2$ p_2 = Some s_2 $\wedge$ match i s_1 s_2

Derived lemma (partial correctness)

context compiler **begin**

$$\frac{\text{compile}\ p_1 = \text{Some}\ p_2 \qquad \text{load}_1\ p_1 = \text{Some}\ s_1 \qquad \text{load}_2\ p_2 = \text{Some}\ s_2 \qquad s_2 \Downarrow b_2 \qquad \neg\ \text{is-wrong}\ b_2}{\exists b_1\ i.\ s_1 \Downarrow b_1 \wedge b_1 \simeq b_2}$$

end

Derived lemma (compiler composition)

$$\frac{\begin{array}{c}\mathsf{compiler} \rightarrow_1 \circledcirc_1\ \mathit{load}_1 \rightarrow_2 \circledcirc_2\ \mathit{load}_2 \sqsubset_{1\text{-}2} \mathit{match}_{1\text{-}2}\ \mathit{compile}_{1\text{-}2} \\ \mathsf{compiler} \rightarrow_2 \circledcirc_2\ \mathit{load}_2 \rightarrow_3 \circledcirc_3\ \mathit{load}_3 \sqsubset_{2\text{-}3} \mathit{match}_{2\text{-}3}\ \mathit{compile}_{2\text{-}3}\end{array}}{\mathsf{compiler} \rightarrow_1 \circledcirc_1\ \mathit{load}_1 \rightarrow_3 \circledcirc_3\ \mathit{load}_3\ [\ldots]\ [\ldots]\ (\mathit{compile}_{2\text{-}3} \Leftarrow \mathit{compile}_{1\text{-}2})}$$

Instantiation example

We used the framework to formalize optimizations of virtual machines.

Three bytecode languages

- **Std** standard (baseline)
- **Inca** inline caching
- **Ubx** unboxed data

Instantiation example: language Inca

datatype *'instr* fundef = ...
datatype (*'fenv*, *'menv*, *'fun*) prog = ...
datatype (*'fun*, *'operand*) frame = ...
datatype (*'fenv*, *'menv*, *'frame*) state = ...
datatype (*'dyn*, *'var*, *'fun*, *'op*, *'opinl*) instr = ...

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locale inca =
  Fenv: env +
  Menv: env +
  dynval +
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locale inca =
Fenv: env F-empty F-get F-add F-to-list +
Menv: env M-empty M-get M-add M-to-list +
dynval is-true is-false +
nary-operation-inl Op Arity InlOp Inl DeInl **for**

F-empty **and** F-add **and** F-to-list **and**
F-get :: *'fenv* ⇒ *'fun* ⇒
(*'dyn*, *'var*, *'fun*, *'op*, *'opinl*) instr fundef option **and**

M-empty **and** M-add **and** M-to-list **and**
M-get :: *'menv* ⇒ *'var* ⇒ *'dyn* option **and**

is-true **and** is-false :: *'dyn* ⇒ bool **and**

Op :: *'op* ⇒ *'dyn* list ⇒ *'dyn* **and** Arith **and**
InlOp **and** Inl **and** DeInl :: *'opinl* ⇒ *'op*

Instantiation example: language Inca (cont.)

context inca **begin**

inductive step :: ('*fenv*, '*menv*, ('*fun*, '*dyn*) frame) state ⇒
('*fenv*, '*menv*, ('*fun*, '*dyn*) frame) state ⇒ bool **where** ...

inductive final :: ('*fenv*, '*menv*, ('*fun*, '*dyn*) frame) state ⇒ bool **where** ...

definition load :: ('*fenv*, '*menv*, '*fun*) prog ⇒
('*fenv*, '*menv*, ('*fun*, '*dyn*) frame) state option **where** ...

sublocale inca-semantics: semantics step final ...
sublocale inca-language: language step final ...

end

Experience report on locales

Framework

- Small code base (1 kLOC)
- Tiny interface (the actual locales)

Instantiations

- Optimization of virtual machines
- Three bytecode languages
- Two compilers
- Somewhat bigger (7 kLOC)

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+ Separation of concerns (parameters, assumptions, derived results)
+ Multiple abstract data types (e.g. *'state*, *'prog*)

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Cons

– Syntactical overhead (predicates, imports, type annotations)
– No parametric types or type aliases in locale definitions
– Different syntax when referencing parameters vs derived definitions

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Parametric data types + absence of type aliases = bloated type annotations.

Overhead grows with the number of locales, parameters, and type variables.
(e.g. Inca-Ubx simulation: 13 type variables, 27 parameters, 11 are shared)

Discussion

What I did (so far)

- Provide complete list of parameters
- Write minimal set of type annotations
- Copy and paste from one locale to the other

Discussion

What would be nice (alternatives)

1. Use locale namespaces to avoid name clashes (as for assumptions)?
2. Use named parameter instantiation (as for lemmas)?
3. Use named type variable instantiation?
4. Use positional type variable instantiation?
5. ???

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2. Use named parameter instantiation (as for lemmas)?
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locale inca =
  Fenv: env[**where** get = F-get] + (1. and 2.)
  Menv: env[**where** get = M-get] + (1. and 2.)
  dynval[**where types** *'dyn* = *'dyn*] + (1. and 3.)
  nary-operation-inl[**of types** *'op 'opinl 'dyn*] **for** (1. and 4.)
    F-get :: *'fenv* ⇒ *'fun* ⇒ (*'dyn*, *'var*, *'fun*, *'op*, *'opinl*) instr fundef option **and**
    M-get :: *'menv* ⇒ *'fun* ⇒ *'dyn* option

Discussion

What would be nice (alternatives)

1. Use locale namespaces to avoid name clashes (as for assumptions)?
2. Use named parameter instantiation (as for lemmas)?
3. Use named type variable instantiation?
4. Use positional type variable instantiation?
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Estimate savings (number of parameters)

inca: 66 % (1. and 2.), 100 % (plus 3. or 4.)
Inca-Ubs simulation: 52 % (1. and 2.), 60 % (plus 3. or 4.) (sharing)

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Thank you

Derived results (behaviour)

context semantics **begin**

$$\frac{\mathsf{step}^*\ s_1\ s_2 \qquad \mathsf{finished\ step}\ s_2 \qquad \mathsf{final}\ s_2}{s_1 \Downarrow \mathsf{Terminates}\ s_2}$$

$$\frac{\mathsf{step}^*\ s_1\ s_2 \qquad \mathsf{finished\ step}\ s_2 \qquad \neg\,\mathsf{final}\ s_2}{s_1 \Downarrow \mathsf{Goes\text{-}wrong}\ s_2}$$

$$\frac{\mathsf{step}^\infty\ s_1}{s_1 \Downarrow \mathsf{Diverges}}$$

end