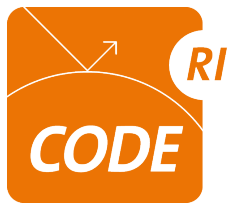
A Generic Framework for Compiler Verification in Isabelle/HOL

Martin Desharnais Stefan Brunthaler



Research Institute
Cyber Defence
Universität der Bundeswehr München

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Isabelle/HOL

Isabelle/HOL is an interactive theorem prover
based on higher-order logic (HOL).

Strengths

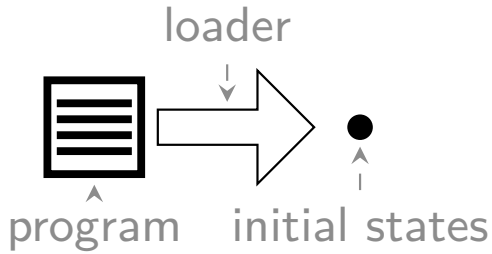
+ small, thrustworthy inference kernel
+ Readable proof format
+ Strong proof automation and tooling
  - general purpose and user defined tactics
  - integration with automated theorem provers (sledgehammer)
  - counterexample generators (quickcheck, nitpick)

This talk focuses on locales: a lightweight module system.

Language semantics

program

Language semantics



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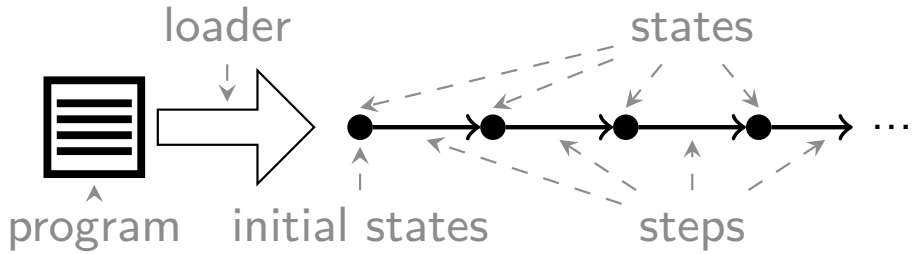

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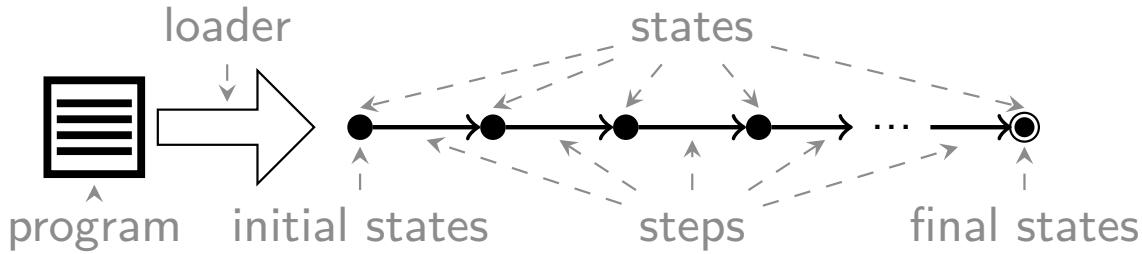

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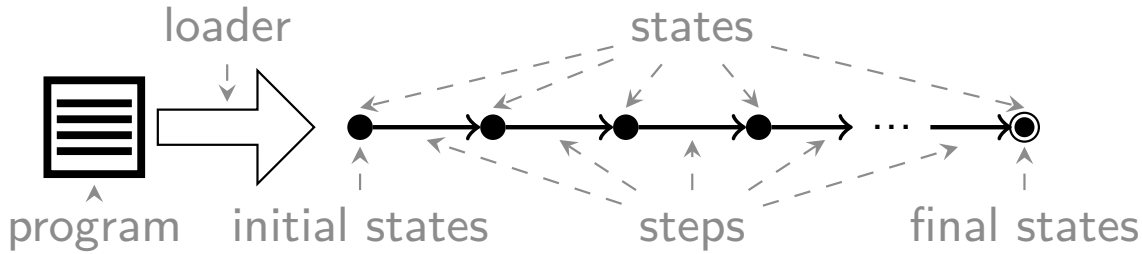

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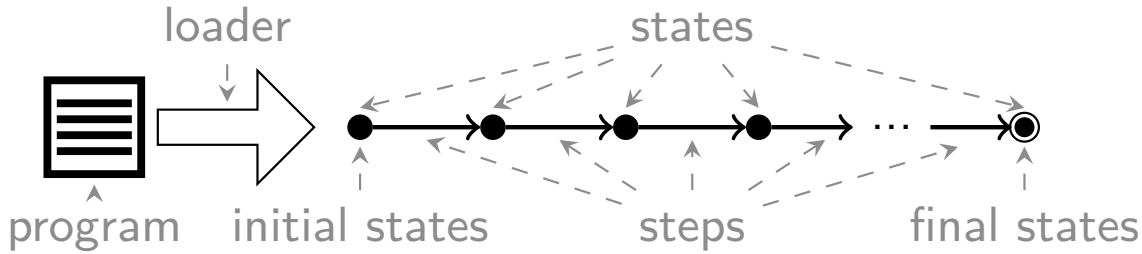

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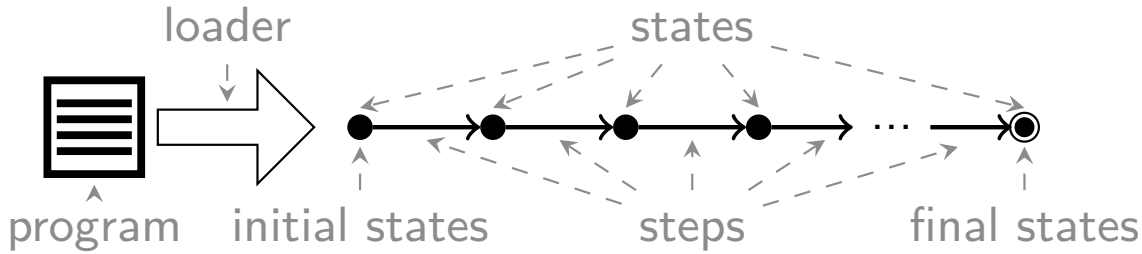

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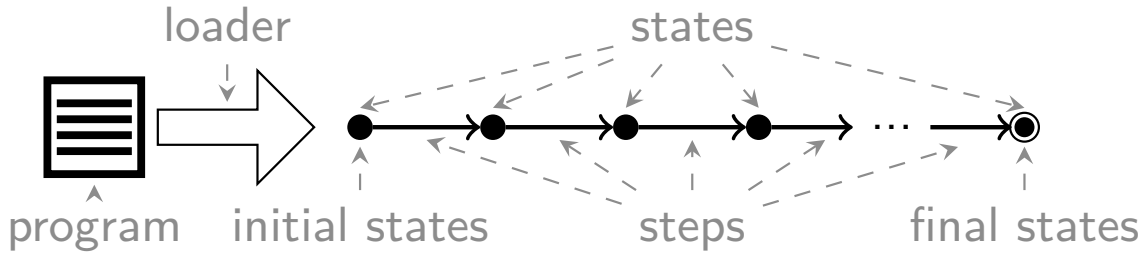

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    → **and** ◉ :: *'state* ⇒ bool +
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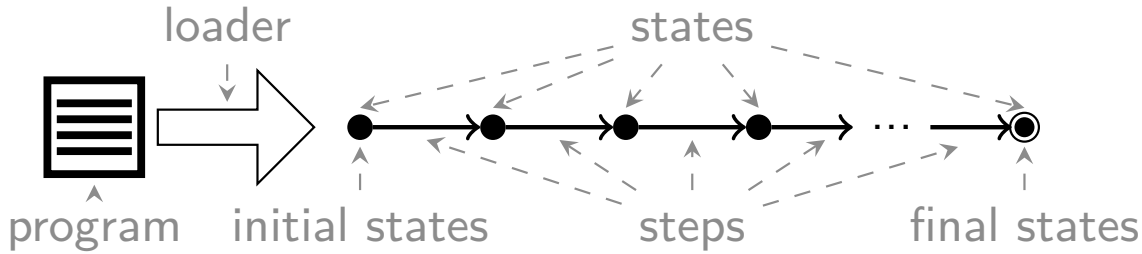

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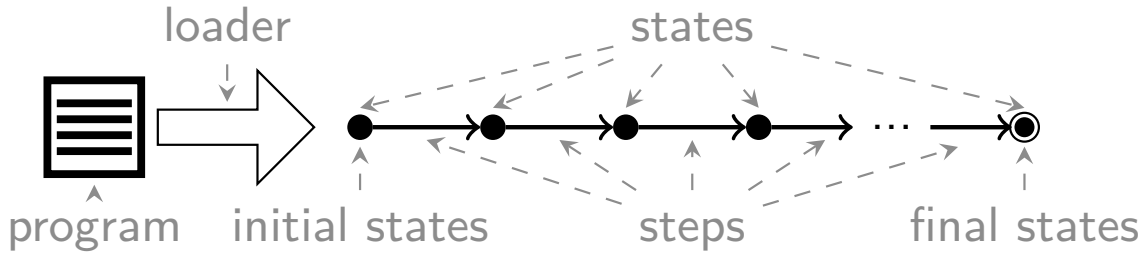

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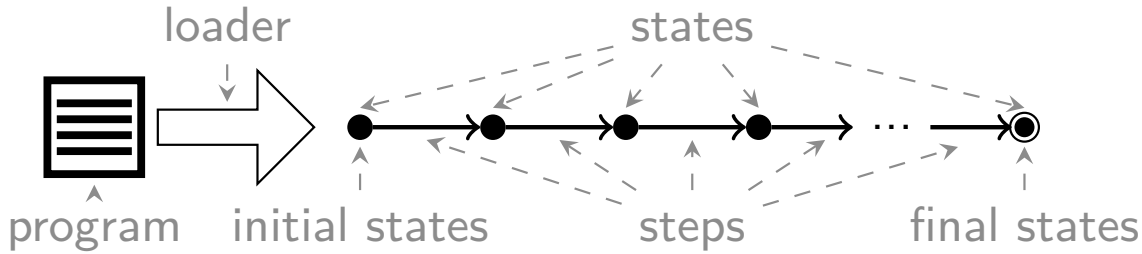

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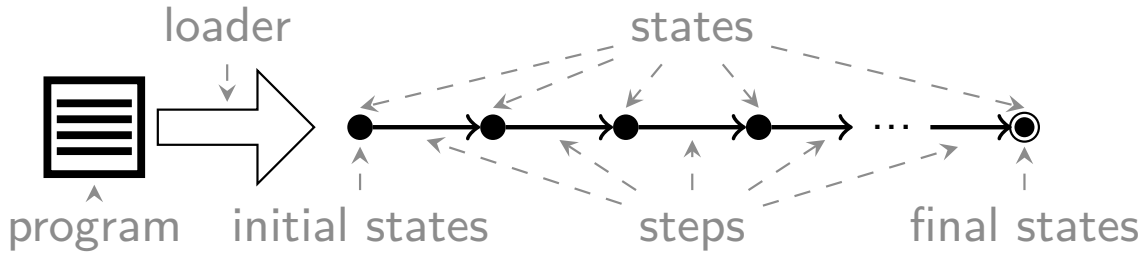

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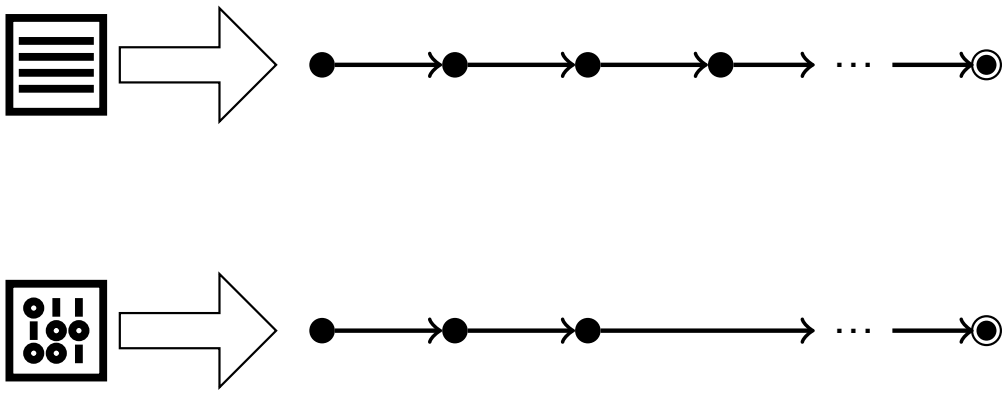
new parameters

Simulation



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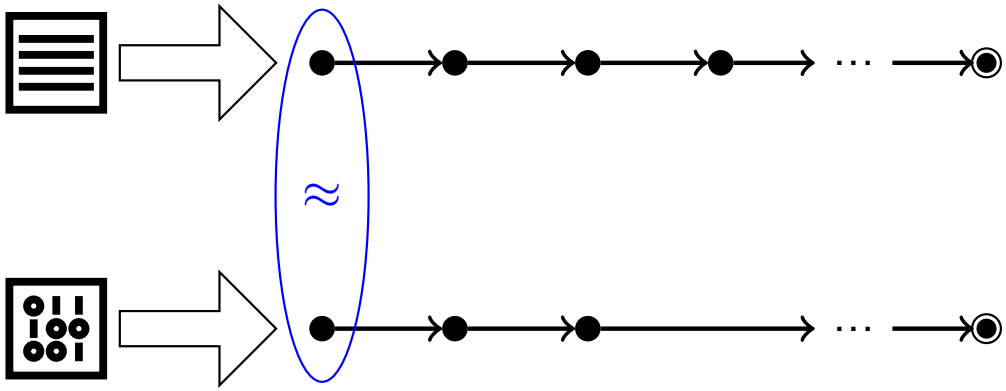

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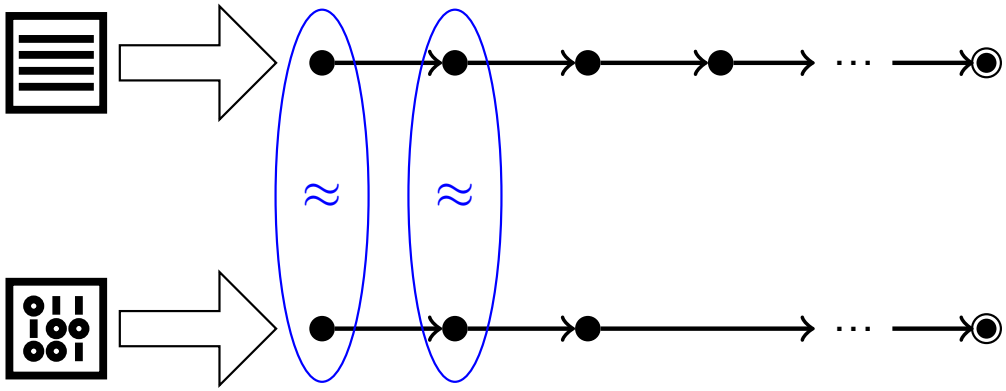

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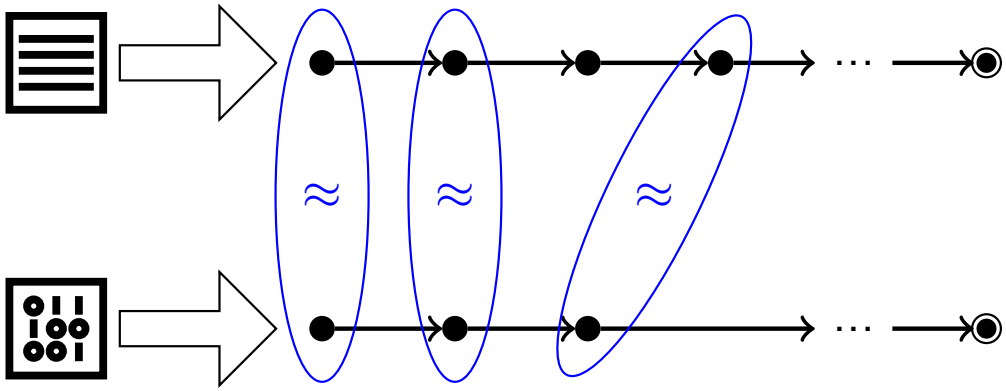

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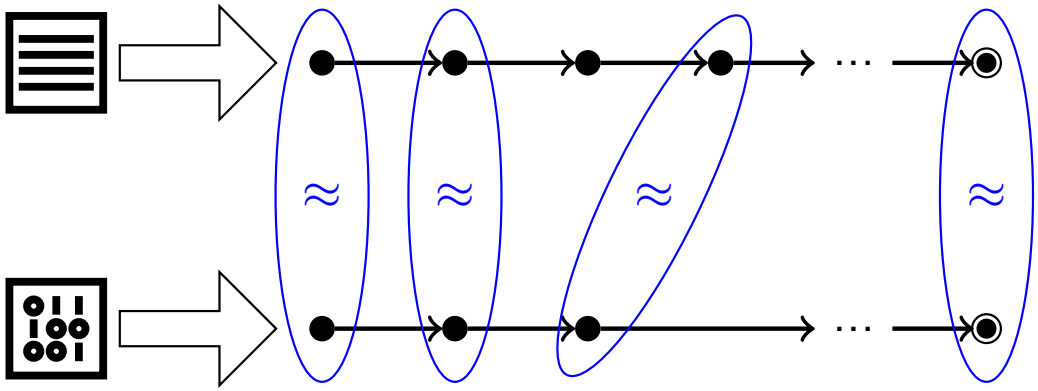

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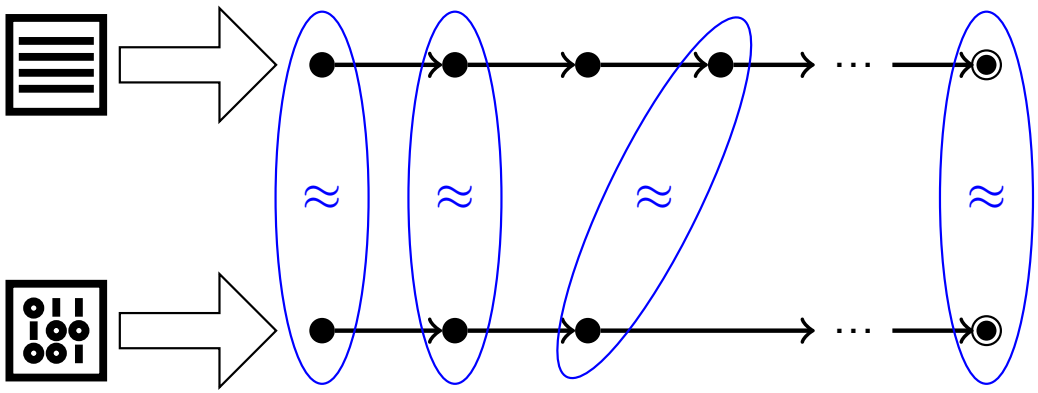

locale backward-simulation =
  L1: semantics $\rightarrow_1$ $\bullet_1$ +
  L2: semantics $\rightarrow_2$ $\bullet_2$ +
  well-founded $\sqsubset$ **for**
    $\rightarrow_1$ **and** $\bullet_1$ :: $'state_1 \Rightarrow$ bool **and**
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    $\sqsubset$ :: $'index \Rightarrow 'index \Rightarrow$ bool +
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  assumes
    match-final: match $i\ s_1\ s_2 \Longrightarrow \bullet_2\ s_2 \Longrightarrow \bullet_1\ s_1$ **and**
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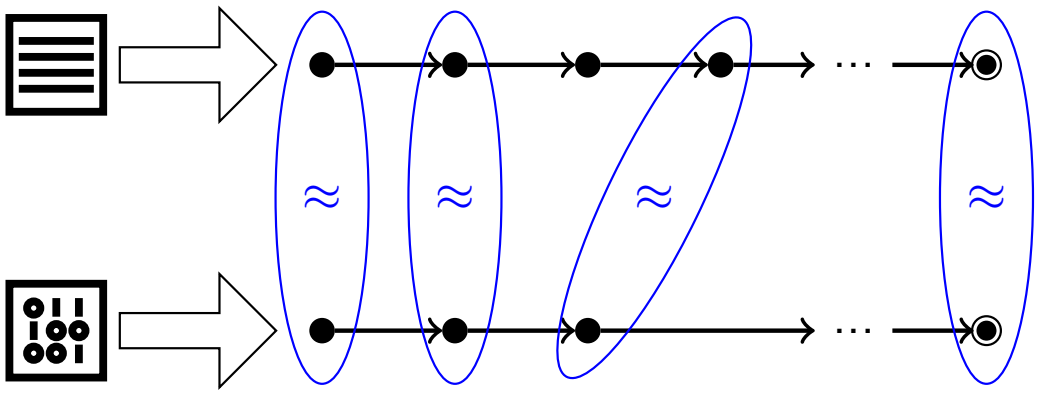

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multiple instances

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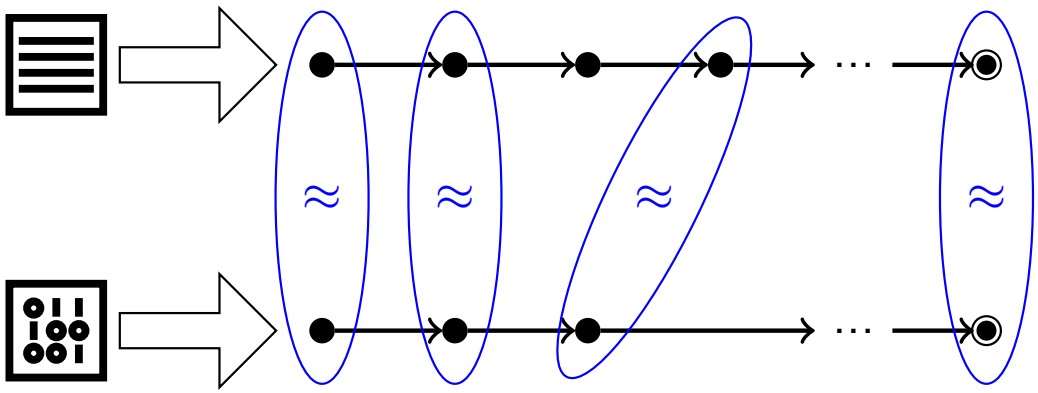

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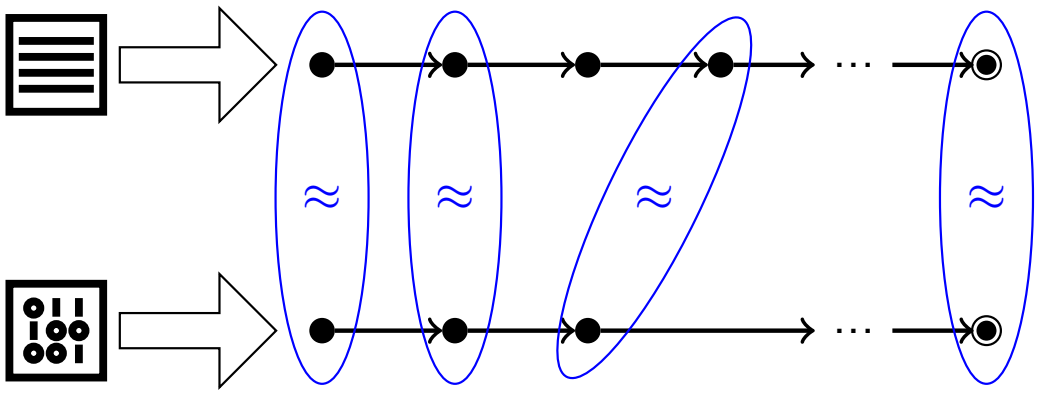

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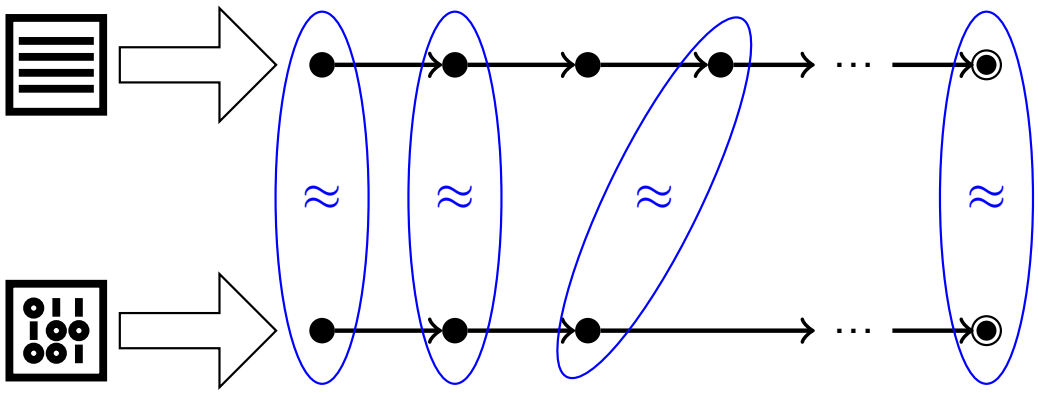

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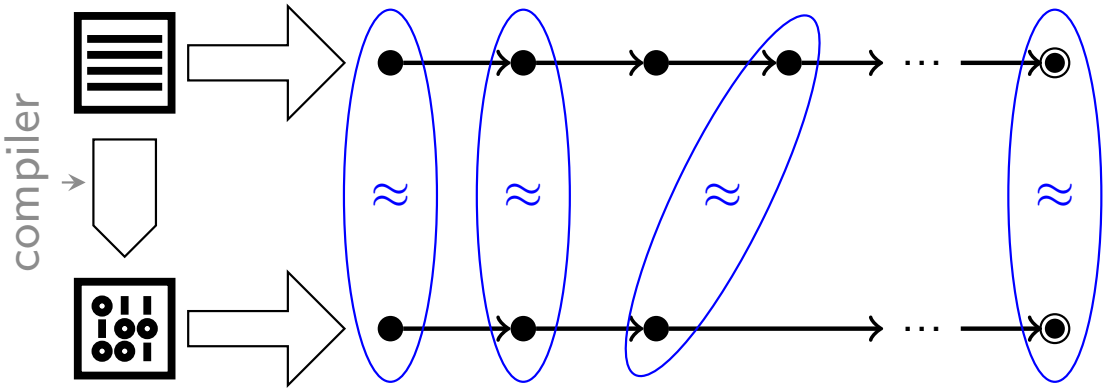
transitive closure

Compiler



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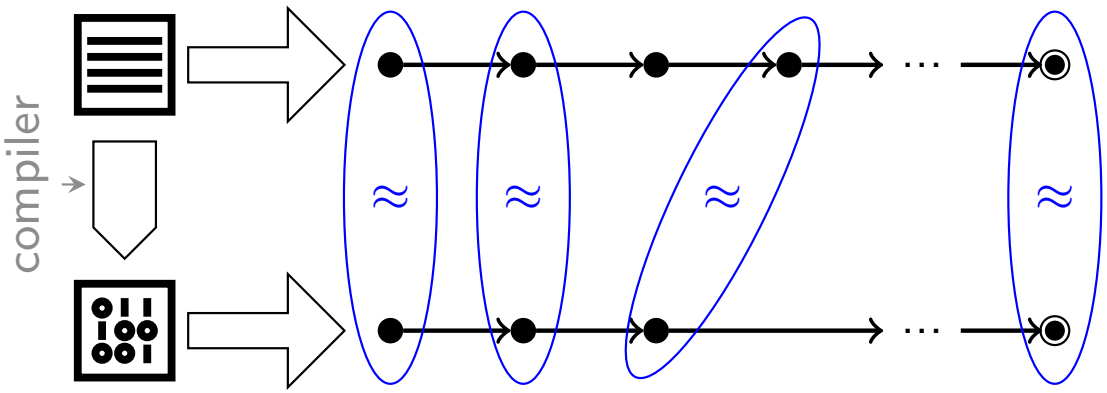

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  backward-simulation $\rightarrow_1$ $\bullet_1$ $\rightarrow_2$ $\bullet_2$ $\sqsubset$ match **for**
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    $\sqsubset$ **and** match
  fixes compile :: *'prog*$_1$ $\Rightarrow$ *'prog*$_2$ option
  assumes
    compile-load: compile p_1 = Some p_2 $\Longrightarrow$ $\exists i$. match i (load$_1$ p_1) (load$_2$ p_2)

Derived lemma (partial correctness)

context compiler **begin**

end

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context compiler **begin**

$$\frac{\text{compile}\ p_1 = \text{Some}\ p_2 \qquad \text{load}_2\ p_2 \Downarrow b_2 \qquad \neg\ \text{is-wrong}\ b_2}{\exists b_1\ i.\ \text{load}_1\ p_1 \Downarrow b_1 \wedge b_1 \simeq b_2}$$

end

Derived lemma (compiler composition)

$$\frac{\mathsf{compiler} \rightarrow_1 \circledcirc_1 \mathit{load}_1 \rightarrow_2 \circledcirc_2 \mathit{load}_2 \sqsubset_{1\text{-}2} \mathit{match}_{1\text{-}2}\ \mathit{compile}_{1\text{-}2} \qquad \mathsf{compiler} \rightarrow_2 \circledcirc_2 \mathit{load}_2 \rightarrow_3 \circledcirc_3 \mathit{load}_3 \sqsubset_{2\text{-}3} \mathit{match}_{2\text{-}3}\ \mathit{compile}_{2\text{-}3}}{\mathsf{compiler} \rightarrow_1 \circledcirc_1 \mathit{load}_1 \rightarrow_3 \circledcirc_3 \mathit{load}_3\ [\ldots]\ [\ldots]\ (\mathit{compile}_{2\text{-}3} \Leftarrow \mathit{compile}_{1\text{-}2})}$$

Experience report on locales

Framework

- Small code base (1 kLOC)
- Tiny interface (the actual locales)

Instantiations

- Optimization of virtual machines
- Three bytecode languages
- Somewhat bigger (7 kLOC)

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Pros

+ Separation of concerns (parameters, assumptions, derived results)
+ Multiple abstract data types (e.g. *'state*, *'prog*, etc.)

Cons

– Syntactical overhead (predicates, imports, type annotations)
– Only first order type variables (limitation of HOL)

Take away

Locales can be **syntax heavy**, but can lead to **generic frameworks** in software formalizations.

Framework users must prove only **three* lemmata**

- all final states match a final state
- there exists a backward simulation
- compilation and loading lead to matching states

to get

- partial correctness
- compositionality

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Thank you

Derived results (behaviour)

context semantics **begin**

$$\frac{\mathsf{step}^*\ s_1\ s_2 \qquad \mathsf{finished\ step}\ s_2 \qquad \mathsf{final}\ s_2}{s_1 \Downarrow \mathsf{Terminates}\ s_2}$$

$$\frac{\mathsf{step}^*\ s_1\ s_2 \qquad \mathsf{finished\ step}\ s_2 \qquad \neg\,\mathsf{final}\ s_2}{s_1 \Downarrow \mathsf{Goes\text{-}wrong}\ s_2}$$

$$\frac{\mathsf{step}^\infty\ s_1}{s_1 \Downarrow \mathsf{Diverges}}$$

end